PHYSICAL MODELING SYNTHESIS: BALANCE BETWEEN REALISM AND COMPUTING SPEED

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ABSTRACT

Physical modeling sound synthesis methods are known to be on the one hand potentially interesting in term of sound richness and probability, but on the other hand expansive to develop and compute.

This paper is dedicated to mass-interaction methods using explicit time. By studying theoretically a specific model, we define a new basic modelization toolkit using non-linear stiffness and we show how it is possible to reach a balance between computing speed and sound realism.

1. INTRODUCTION

Physical modeling is increasingly used to synthesize sounds: it allows both interesting sound realism and new synthesis controls [1]. Nevertheless, it causes a computing speed problem no matter which modelization principles we use. To simulate a physical model and synthesize a rich audio signal, a significant number of state variables are needed. Moreover, the calculations computed at each step to define the new state are complex, and use expensive functions in term of processing time.

Therefore, if the aim is not only a precise modeling but also to create a general physical synthesis environment, computing speed has to be increased as much as possible.

There are several ways to achieve this. Algorithm optimization is needed of course, but it is useful to work on and simplify the modeling bases and modeling principles themselves. A balance between synthesized sound realism and computing speed then has to be chosen.

In this paper we emphasize explicit time mass-interaction models - and particularly their spatiality, the way they are initially built and then move in the 3D virtual space. Firstly we explain how to describe spatiality in mass-interaction models and we study algorithm complexity of 3D models. We explore then an example that shows the interest of spatiality in sound. A theoretical study of this model leads us to define a new 1D-modeling base that presents an interesting balance, and to implement it within the CORDIS-ANIMA system. The choices we carried out to allow a relevant control of the new parameter of 1D models are then explained.

2. MODELIZATION COMPONENTS

We study within the paper explicit time mass-interaction models, which modularity and genericity are of interest to define a whole physical synthesis environment.

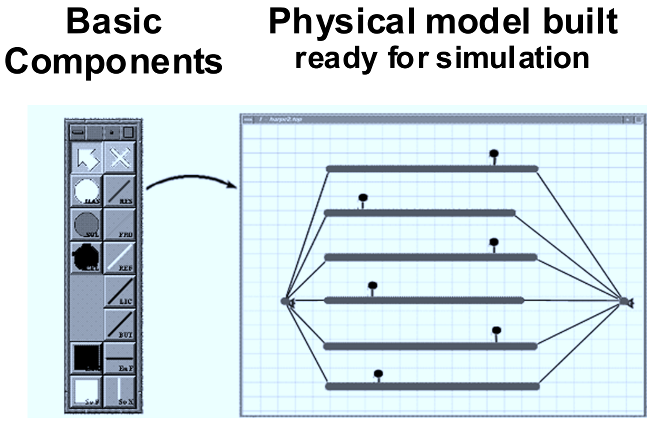

Fig 1: *building a model*

Such a model is built connecting basic elementary components (or “modules”), mainly masses and linear visco-elastic interactions.

Other elementary modules can be used: fixed points (infinite inertia masses), buffer interactions (effective only while the two connected masses are inside a mutual interaction field), other complex interactions implementing hysteretic behaviors…

Once parameters (physical and initial conditions) are given to every elementary module used, the model can be simulated.

Simulation uses explicit time methods, with a given frequency (44100 Hz usually). The new state (position, speed…) of each mass at the moment t=n is found computing the new forces (using state at t=n-1), then the new position.

To extract sounds from a moving model, we write in a file the position of one of the masses among one of the axis. The samples thus created can then be sent on speakers.

3. DESCRIBING SPATIALITY OF A MODEL

To describe a model, it is necessary to explain the topology of its connection network ([5], i.e. how its N masses are inter-connected through L interactions) and to give the parameters used, but also to describe its spatiality. As we shall see, choice of a spatiality is significant for sound quality and computation speed.

In order to describe efficiently a spatiality model we were led to define three spaces. Given a topology (number of masses, springs and other basic components, connection network), we distinguish the spatial appearance, the space of the movements and the simulation space.

3.1.1. Spatial appearance

The way a model occupies its space before simulation is called the spatial appearance.

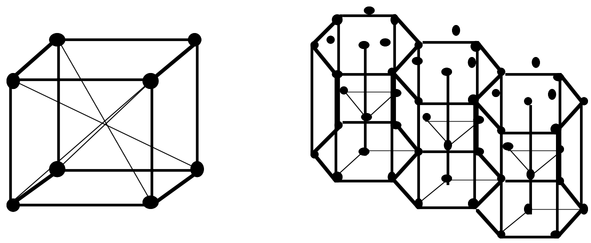

Figure 2: *3D-appearance models*

Based on the spatial appearance we can define the kind of gesture (dynamic In data flow that control one of the masses' position throughout simulation) the model can receive and some of its oscillatory properties; the spatial appearance allows to distinguish wave directions, such as "transversal" and "longitudinal".

A model with a 0D-spatial appearance can be defined. In such a model, before excitation, all the masses are at the same position.

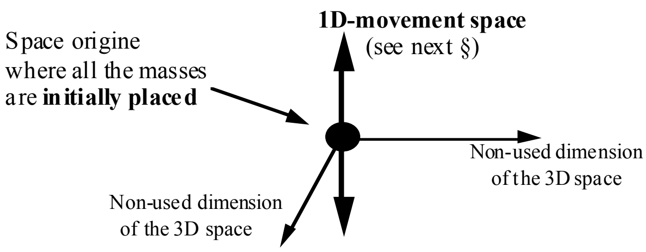

Figure 3: *a 0D-appearance model...*
with 1000 masses and springs !
Here transversal and longitudinal waves cannot be distinguished.

3.1.2. Space of the movements

The space of the movements is the space among which the masses can move. Its dimension is proportional to the model's number of dynamic variables.

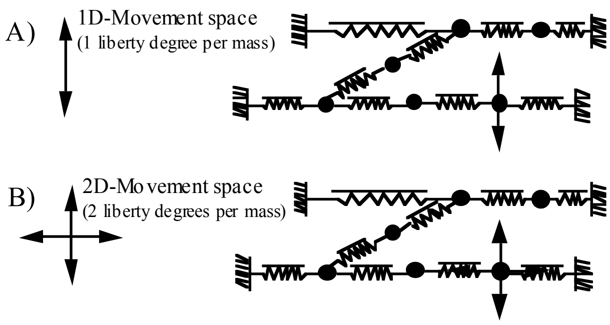

Figure 4: *a 2D-qppearance model*
top (A): with a 1D-movement space
bottom (B): with a 2D-movement space

In a 1D-movement model, only three variables have to be computed on each step per mass: position, speed and force. In a 2 or 3D-movement model with the same number of masses and springs, the number of calculations needed doubles or triples.

3.1.3. Simulation space

We call simulation space the mathematical summon of spatial appearance and movement space. It is the minimum space that has to be defined in the program in order to simulate the model.

The simulation space implies the mathematical distance used to perform the simulation.

As a consequence, the dimension of the simulation space implies the linearity or non-linearity of the model's equation. A 1D-simulation space model built with masses and springs is a linear model because the 1D distance is linear and thus the equations that define its movements are linear. 2D or 3D-simulation space models are no more linear.

Models with a complex spatiality do present, as we shall see, some sound specific interest, but needs an important computing time due to their algorithm complexity. The next paragraph studies the algorithm complexity of such spatially complex models.

4. ALGORIYHM COMPLEXITY OF 3D MODELS

The complexity study of the algorithms for a 3D-simulation space model (each mass has 3 freedom degrees) built with N masses and L spring-frictions leads the following results:

- To obtain a rich sound timbre, N is important
- The state system is described with at least 2*3N variables: position and delayed position of each mass on the three axes. These variables have to be calculated on every simulation step.
- L distances are calculated for each step, so that the sqrt function is called L times - L is often greater than 1000.
- At each step, the calculations needed are about :
  - *(30*N + 14*L) additions*
  - *(9*N + 8*L) multiplication*
  - *L calls to the sqrt function*
  - *(6*N + L) assignments*

Simulating is time expensive, mainly because of distance calculations and the freedom degrees for the masses, i.e. because of model spatiality.

5. STUDY OF A SPATIAL STRING-LIKE MODEL

Studying a specific model and listening to its sound one can show how using spatiality in physical modeling is of interest for the ear.

5.1. The experimentation model

The string-like model we used to study the sound potentiality of spatiality is made of 50 masses. Each mass has two freedom degrees and is connected to the next one with a visco-elastic interaction. The "string" is attached at its extremities and receives a pre-stressed tension. In term of spatiality, it is a 1D-spatial appearance, 2D-movement space and 2D-simulation space model.

The parameters of the model are:
- *M the inertia of each mass*
- *T the pre-stressed tension applied one its extremities*
- *Lp the length of a spring under the effect of the pre-stressed tension*
- *K qnd Z the stiffness and damping of the springs.*

In order to excite the model a percussive excitator built with a mass and a buffer visco-elastic interaction is launched on one mass of the string. The sound is made during the simulation writing on each step the position among the Z-axis of the 10^{th} mass in a floating point sound file.

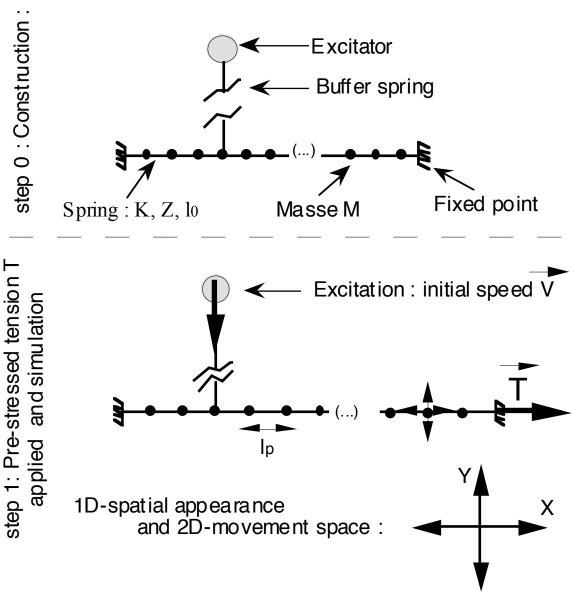

Figure 5: *the 2D string-like model*

The string-like model equations are not linear, because the 2D distance calculations call the sqrt function:

$$d(mass1, mass2) = sqrt((x_1 - x_2)^2 + (y_1 - y_2)^2)$$

and this physical non-linearity can be heard in sound.

5.2. Non-linear sound effects in the spatial model

The spatial non-linearity is not perceptible with insufficient excitation levels, but becomes hearable when excitation increases. Then, two categories of sound effects are found according to the physical parameters used (inertia, stiffness...).

5.2.1. First non-linear sound effect of spatiality

In small parameter fields, a modulation of the fundamental frequency around its medium value rises. This case is shown off on the next sonogram.

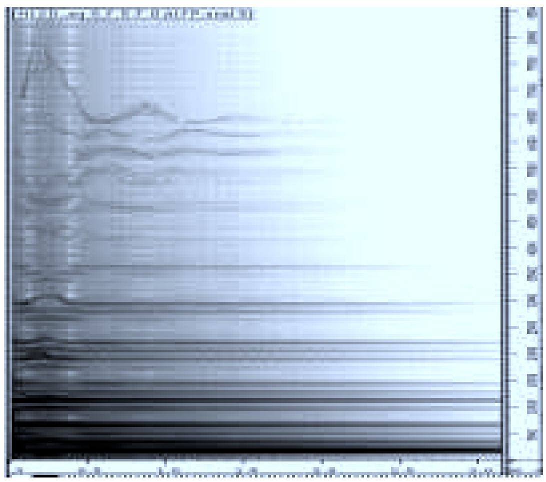

Fig 6: *sonogram of a model that shows off the first non-linearity*

This non-linear effect appears when the longitudinal and transversal modes are strongly coupled and when the longitudinal fundamental mode is at low frequency, i.e. in term of parameter when the springs' stiffness is small compared to masses' inertia.

The next figure is a picture of a model with such a non-linear behavior.

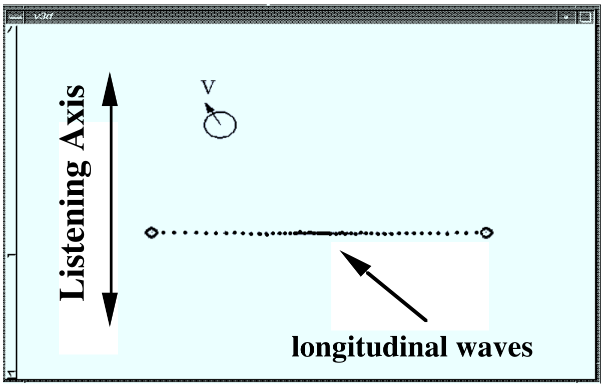

Fig 7: *picture of a model that shows off the first non-linearity during simulation*

In this model, the low frequency longitudinal waves modulate strongly the local transversal stiffness. Actually, this modal cooperation behavior appears in all the 2D String-like models; but it becomes hearable only for a few parameters set of values.

5.2.2. Second non-linear sound effect of spatiality

The second most common and most important effect is a slide of the fundamental (as shown in the next sonogram) with at the beginning, a possible hearable sound saturation

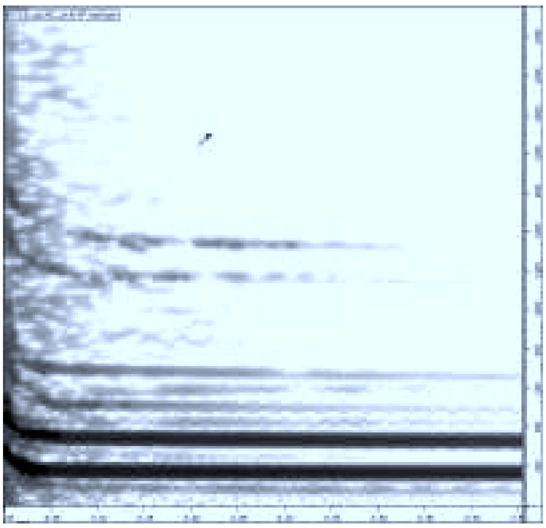

Fig 8: *sonogram of a model that shows off the second non-linearity*

This non-linear effect appears when the length of the string is strongly increased while it moves.

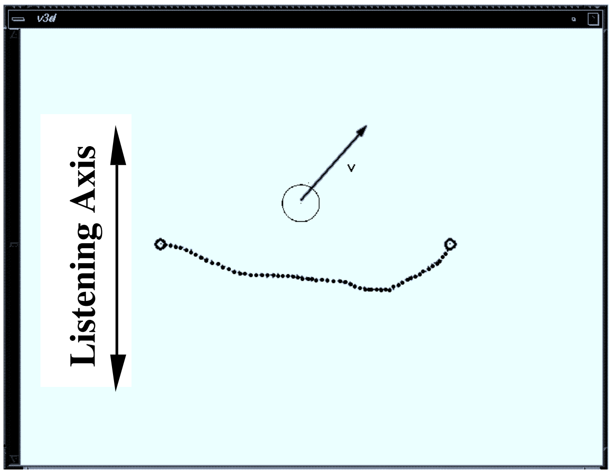

Fig 9: *picture of a model that shows off the second non-linearity during simulation*

This second effect is strongly correlated to the excitation level, i.e. the excitator's speed. Its physical origins are explained in many studies of membrane and string behavior [2] [3]. It is due to an increase in the stiffness forces with the string-like model lengthening, right after percussion.

This excitation-correlated sound effect is known to be important in natural sounds: the ear analyses it as the excitation level signature. Thanks to it, the ear can guess that a natural sound was made by a strongly excited object even if this sound is played or heard at a very low loudness.

5.3. theoretical simplification approach: 1D "equivalent" model

In most case when a model present the second non-linear effect the movements are mainly transversal and one can say that the longitudinal movements are mathematically negligible – which is proved both by measures and theoretical study.

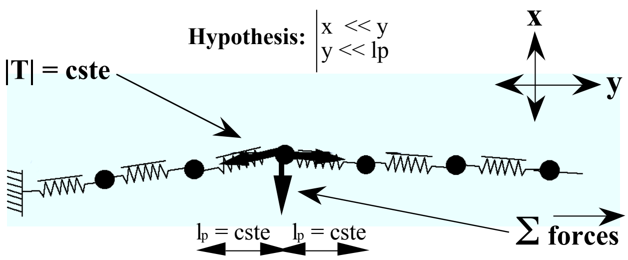

Fig 10: *Simplifications hypothesis*

Considering that the resulting force on a mass has no effect on movement, the force vector becomes transversal (such as the movements).

Now we can develop the transversal force using two mathematical hypotheses

- $(x\text{-}x_0) << y$ hyp. 1 "movements are transversal"
- $y << lp$ hyp. 2 "small deformations"

On the second order, we are led to the following results:

$$\boxed{\begin{array}{c} F_{n+1->i,Y} = \alpha * DeltaY_n + \beta * DeltaY_n^3 + o((DeltaY_n)^4)) \\ \textit{where} \\ \alpha = -\frac{T}{l_p} \\ \textit{and} \\ \beta = -\frac{K}{2*l_p^2} + \frac{T}{2*l_p^3} \end{array}}$$

where T, K, and lp are the model parameters and $deltaY_n$ is the transversal distance between the two masses at t=n.

This equation has only one dimension, and it contains:

- a first linear term $(\alpha * deltaY_n)$, which does look like a 1D stiffness
- a non-linear term that modulate the stiffness.

Based on it, we can try to built a new 1D-simulation space string-like model using a non-linear stiffness.

6. A 1D NON-LINEAR STRING-LIKE MODEL

According to the precedent theoretical development we defined another string-like model but with a 1D-simulation space and non-linear interaction. This model is shown on the next figure.

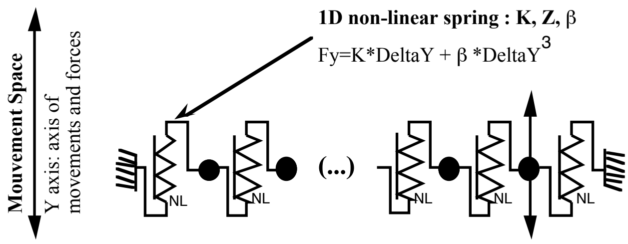

Figure 11: *the 1D-simulation space equivalent model*
It is a 0D-appearance model
(here 2D-viewed for representation reasons)

Experimentally, this non-linear model presents the second non-linear sound effects we found of a main interest in the 2D model, while it is height times quicker to simulate. The sound richness is a bit less than with the 2D model, but it still presents the most interesting excitation-dependant psycoacoustic characteristic.

7. A 1D NON-LINEAR ELASTICITY

The study above leads us to define a new modeling base for 1D-space movement models. Before introducing this new module, let us expose the 1D linear models' algorithm complexity.

7.1.1. Short study of 1D-simulation space linear models

Because the distance between two masses is, in such models, a linear function of the masses positions on the movement axis Y and because only one variable describe a mass position, a few calculations are needed for each step for a N masses and L visco-elastic interaction model:

*(N+5*L) additions*
*(N + 2*L) multiplications*
*(2*N+K) affectations*

Thus, a 1D linear model with a given (N, L) complexity is much easier to simulate than a 3D one - but the sound may be of less value.

7.1.2. Definition of a non-linear stiffness

For 1D models we define a new stiffness algorithm that needs two parameters K and λ. In this case the force becomes:

$$F_n = K * (1 + \lambda * deltaY_n^2) * deltaY_n$$

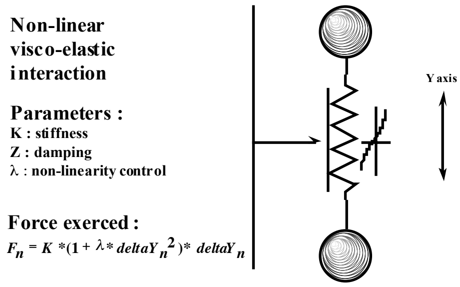

Figure 12: *the new basic interaction*

This stiffness force is a non-linear function of the two masses' position $deltaY_n$, with connection to the forces studied above.

To simulate a model with N masses and L non-linear visco-elastic interactions, for each step the computer needs to calculate:

(N+6*L) additions
*(N + 4*L) multiplications*
*(2*N+K) affectations*

This is much less than for a 3D model with the same (N, L) complexity, and close to a simple 1D linear model.

The sounds produced by models built with the new non-linear elasticity present the second effect found in the 2D "string" sound. Timbre of non-linear 1D models depends on excitation level; the ear actually analyzes this timbre modification as a sound image of the excitation level.

8. CONTROLING THE NON-LINEAR EFFECT

Because important models cannot be real-time simulated (even though their simulation space is one-dimensional), it is of interest to draw out a process that allow a control the non-linear effect parameter.

In order to find a relevant control of this new parameter, we studied first 1D-simulation space string-like and surface-like kinds of models built with the new non-linear stiffness.

A preliminary study proved that the maximum value of $(\lambda * deltaY_n^2)$ within all the springs of the model and throughout the simulation gives a good measure of the non-linear effects that can be heard in sound.

According to this result, a relevant process was drawn in order to propose to users a control of lambda.

Two methods are possible:

The first one is a simple control of the value of lambda, which is to be given by the user.

The second supposes a linear 1D model (with linear springs) to be designed first, with its initial conditions and its gestures (and so on its medium excitation level). The process to give a relevant value to lambda is then (see also Fig. 13):

- 1) To ask the user to give a NL coefficient between 0 and 1, describing the non-linearity he would like to perceive in the sound (NL = 0 <=> no non-linearity; NL =1 <=> important non-linearity).
- 2) To run a first simulation on the linear model in order to calculate the maximum value DeltaYmax of $deltaY_n$ within all the springs, during the whole simulation (excitation level estimation).
- 3) To calculate lambda as $\lambda = NL / deltaY\max^2$

Such a process allows an efficient control of the non-linear effects that will be perceived in the synthesized sound.

9. A NEW CORDIS ANIMA MODELISATION TOOLKIT: IMPLEMENTATION WITHIN GENESIS

CORDIS-ANIMA [4] is the physical-modeling language that has been developed and increased since 1975 in ACROE-ICA laboratory in Grenoble, France. Its graphically interfaced implementation, GENESIS, is a sound synthesis environment program that allow composers and researchers to define 1D mass-interaction physical models, control in different ways their parameters, and simulate them synthesizing sounds.

The 1D non-linear elasticity algorithms we drew out was added to the CORDIS-ANIMA basic component toolkit in the 1.4 GENESIS release by the beginning of September. It allows the definition of non-linear models which sound and timbre are correlated to excitation level and that are still quick to simulate. Composers will test this new basic interaction component during the next months.

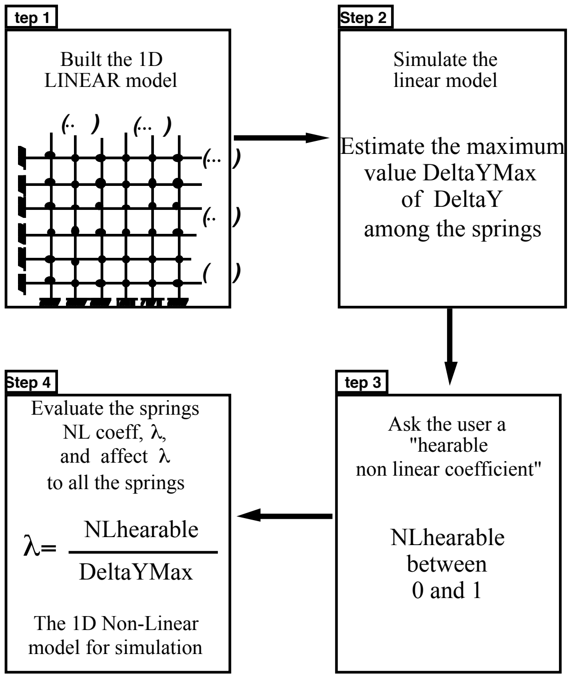

Fig 13: *control of the non-linear parameter λ*

10. CONCLUSION

Working within the physical mass-interaction principles, we found a new interesting tradeoff between time processing and sound realism.

We added to the 1D modeling base a 1D elementary non-linear stiffness that produces the sound effect we were looking for, but at a low price in term of processing time. The natural non-linear dependence between sound level and timbre it allows, known to be important in real structures sounds, opens new ways of sound synthesis.

11. REFERENCES

[1] Risset, J.C., “Modèle physique et perception - Modèle physique et composition“ - *Colloque International Modèles Physiques Création Musicale et Ordinateur Vol.III (p711-720) - Grenoble France, 1990.*

[2] Watzky A. , “Vibration des cordes en régime non-linéaire : modèle bidimensionnel“ - *Premier congrès français d'acoustique, supplément au Journal de Physique, tome 2, 1990.*

[3] Cuesta, C., “Corde vibrante isolée mécaniquement, amortissements, non-linéarités ; application au clavecin et à la tempoura“ - *thèse soutenue à l'université du Maine le 9 Mars 1990.*

[4] Cadoz, C., “Sound, Music and Image Creation with physical model - The CORDIS and GENESIS tools from ACROE“, *DAFx00 Workshop, 99.*

[5] Incerti E, Cadoz, C “Modélisation et simulation de structures vibrantes en *CORDIS*; Matériaux et paramètres pour la création musicale” - *Journées d'Informatique Musicale, 1995.*